1. Exercise: Short channel MOSFETs

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1. Exercise: Short channel MOSFETs (Solution)

- Calculate $K'$

\[ K_n' = \mu_n C_{OX} \quad \quad K_p' = \mu_p C_{OX} \]

<table>
<thead>
<tr>
<th></th>
<th>$V_D$ [V]</th>
<th>$K'$ [A/V²]</th>
<th>$\mu$ [cm²/Vs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMOS</td>
<td>0.4</td>
<td>115* 10⁶</td>
<td>$\mu_n = 1.15* 10^4$</td>
</tr>
<tr>
<td>PMOS</td>
<td>-0.4</td>
<td>30* 10⁶</td>
<td>$\mu_p = 3.00*10^3$</td>
</tr>
</tbody>
</table>

1. Exercise: Short channel MOSFETs (Solution)

- What is the value of $\kappa$ for a long channel MOSFET?

$\kappa$ can be derived from the following equation:

\[ \kappa(V_{DS}) = \frac{1}{1 + \frac{V_{DS}}{E_c L}} \]

For a long channel MOSFET, $\kappa$ can be assumed as 1 and so be neglected.

1. Exercise: Short channel MOSFETs (Solution)

- Estimate the drain current $I_{DS}$ for both MOSFETs in ohmic region using the classical expression and using the velocity saturation effect. Compare both results by calculating the percentage of error between the results.

The classical Expression for the drain current of a N-channel-MOSFET is:

\[ I_D = \frac{\mu_n C_{ox}}{K_n L} \left( \frac{W}{L} \right) \left( V_{GS} - V_T \right) V_{DS} - \frac{V_{DS}^2}{2} \]
1. Exercise: Short channel MOSFETs (Solution)

The drain current including the velocity saturation effect is:

\[ I_D^{(sc)} = \kappa(V_{DS}) \cdot K_n \left( (V_{GS} - V_T) \cdot V_{DS} - \frac{V_{DS}^2}{2} \right) \]

with: \( \kappa(V_{DS}) = \frac{1}{1 + \frac{V_{DS}}{E_c L}} \)

For the error we get:

\[ \varepsilon = \frac{I_D - I_D^{(sc)}}{I_D^{(sc)}} = \frac{1 - \kappa(V_{DS})}{\kappa(V_{DS})} = \frac{1}{\kappa(V_{DS})} - 1 = \frac{|V_{DS}|}{E_c L} \]

• Taking the values given above, we get for the NMOS-Transistor:

\[ I_D = 115 \times 10^{-6} \cdot \frac{0.75 \, \mu m}{0.25 \, \mu m} \left( (0.6 \, V - 0.4 \, V) \cdot 0.1 \, V - \frac{(0.1 \, V)^2}{2} \right) = 5.175 \, \mu A \]

• and for the PMOS-Transistor:

\[ I_D = 30 \times 10^{-6} \cdot \frac{0.75 \, \mu m}{0.25 \, \mu m} \left( (-0.6 \, V - (-0.4 \, V)) \cdot (-0.1 \, V) - \frac{(-0.1 \, V)^2}{2} \right) = 1.35 \, \mu A \]

• Calculating \( \kappa \) for NMOS and PMOS device yields:

\[ \kappa(V_{DS}) = \frac{1}{1 + \frac{|V_{DS}|}{1.5 \, \mu m \cdot 0.25 \, \mu m}} = 0.79 \]

• Therefore we get for the NMOS device:

\[ I_D^{(sc)} = 0.79 \cdot 5.175 \, \mu A = 4.1 \, \mu A \]

• and for the PMOS-Transistor:

\[ I_D^{(sc)} = 0.79 \cdot 1.35 \, \mu A = 1.1 \, \mu A \]

• The error for both devices is:

\[ \varepsilon = \frac{0.1 \, V}{1.5 \times \mu m \cdot 0.25 \, \mu m} = 27\% \]

These short channel device enters saturation 0.07V before their \( V_{DS} \) reaches 0.2V as is predicted by the classical assumption:

\[ V_{DSAT} = V_{GS} - V_T = 0.6 \, V - 0.4 \, V = 0.2 \, V \]
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- Find an expression for the on-resistance of short channel devices and estimate the on-resistance for both devices.

The on-resistance is by definition:

\[ R_{on} = \lim_{V_{DS} \to 0} \frac{1}{g_{DS}} \]

with:

\[ g_{DS} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{DS} \to 0} \]

For the short channel devices:

\[
R_{on}^{(s)} = \left( \frac{\partial I_D^{(s)}}{\partial V_{DS}} \right)^{-1} \bigg|_{V_{DS} \to 0} = \left( \frac{\partial \kappa(V_{DS})}{\partial V_{DS}} \cdot I_D + \kappa(V_{DS}) \cdot \frac{\partial I_D}{\partial V_{DS}} \right)^{-1} \bigg|_{V_{DS} \to 0}
\]

Therefore the on-resistances of a short and a long channel device are equal. This is due to the fact, that at small \( V_{DS} \) the carriers are not velocity saturated!

Finally we get:

\[
I_D = K \left( (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)
\]

\[
\left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{DS} \to 0} = K \left( (V_{GS} - V_T) - V_{DS} \right)_{V_{DS} \to 0}
\]

\[
= K \left( V_{GS} - V_T \right)
\]

Only positive values for \( R_{on} \) make sense. Therefore we get for NMOS and PMOS device:

\[
R_{on}^{(NMOS)} = 14.5 \text{ k}\Omega \quad R_{on}^{(PMOS)} = 55.5 \text{ k}\Omega
\]